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MIXING OF $\nu_e - \nu_\mu$ IN SO(10) MODELS

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ABSTRACT

Because of the hierarchy of quark masses, $\nu_e - \nu_\mu$ mixing in SO(10) is generally small. We illusrate quantitatively how the parameter characterizing the Majorana sector must be tuned in order to achieve large $\nu_e - \nu_\mu$ mixing.

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It has been found in the framework of SO(10) grand unified field theories[1] with the Gell-Mann-Ramond-Slansky (GRS) mechanism[2] generating neutrino masses that no significant mixing of ν_e with other neutrinos occurs, while ν_μ-ν_τ mixing can be substantial.[3,4] There are, however, experiments suggesting a non-zero ν_e mass and mixing.[5] As a result, it is interesting to pursue further the possibility of ν_e mixing in this framework.

We examined elsewhere a novel model with two Higgs suggested by Glashow[6] and found[7] in the optimum case that ν_e-ν_μ mixing can be of the order of the Cabibbo angle θ_c. To illustrate the difficulty of achieving large ν_e-ν_μ mixing, let us extend this analysis by using the Witten mechanism[8] for generating the Majorana masses, which gives an additional degree of freedom (as can be seen below). The same freedom can be obtained by using three Higgs in the GRS mechanism. We use the former language to give a simple one-parameter characterization of the Majorana masses. In fact, we can find, for two generations, large mixing only for an extremely narrow range of this parameter. We expect this to be a general feature.

In this model[6] there are two Higgses of SO(10), **10** and **126**, which generate the down quark, up quark, charged lepton, Dirac neutrino, and Majorana neutrino mass matrices, respectively, as follows:

$$M^D = a_1 F^{10} + b_1 F^{126}, \tag{1}$$

$$M^U = a_2 F^{10} + b_2 F^{126}, \tag{2}$$

$$pM^L = a_1 F^{10} - 3b_1 F^{126}, \tag{3}$$

$$p\nu = a_2 F^{10} - 3b_2 F^{126}, \tag{4}$$

$$pM = a'_3 F^{10} + b'_3 F^{126}. \tag{5}$$

The coefficients a_i, b_i are vacuum expectation values of the Higgs bosons, and p is the renormalization group correction coming from the extrapolation from 10^{15} GeV to 10^2 GeV. The matrices F^{10} and F^{126} represent the Yukawa couplings among the families to the **10** and **126** of Higgs, respectively. Here in the Witten mechanism,[8] the Majorana mass of the right-handed neutrino is zero at the tree level but is generated by a radiative correction at the two loop level. Note that in Eq. (5), both **10** and **126** contribute to the Majorana mass matrix because **10**×**45**×**45** (where the gauge bosons are in the **45**) and **126**×**45**×**45** contain a **126** representation which develops a vacuum expectation value along the SU(5) singlet direction.[8] These vacuum expectation values are estimated to be[8]

$$a'_3/a_2 \sim b'_3/b_2 \sim (\alpha/\pi)^2(M/M_W),\ M \sim 10^{15},\ M_W \sim 10^2 \text{ GeV}. \tag{6}$$

We find from Eqs. (1), (2), (4), and (5)

$$p\nu = [-(3a_1b_2 + a_2b_1)M^U + 4a_2b_2 M^D]/(a_1b_2 - a_2b_1), \tag{7}$$

$$pM = (a_1b'_3 - a'_3b_1)/(a_1b_2 - a_2b_1)[M^U - \beta M^D], \tag{8}$$

where we assume $a_1b_2 - a_2b_1 \neq 0$, and

$$\beta = (a_2b'_3 - a'_3b_2)/(a_1b'_3 - a'_3b_1). \tag{9}$$

Because of the large disparity between the Majorana mass scale and the quark mass scale, we need only diagonalize the matrix

$$\mathcal{M} = -\nu M^{-1}\nu \tag{10}$$

in order to determine the left-handed neutrino masses and mixings.

Let us begin by considering the first two generations of fermions, (u, d, e, ν_e) and (c, s, μ, ν_μ). The only possible mass matrices that can be derived from a discrete symmetry with mass relations[9] $d/s = 9e/\mu$ have the SO(10) structure,

$$M^U = \begin{pmatrix} 0 & \mathbf{10} \\ \mathbf{10} & \mathbf{126} \end{pmatrix} = \begin{pmatrix} 0 & (uc)^{1/2} \\ (uc)^{1/2} & c \end{pmatrix}, \tag{11}$$

where letters represent the quark masses and M^D can be obtained by the substitution $(u,c) \to (d,s)$ in (11). From (1), (2), and (11) we deduce

$$\begin{aligned} a_1/a_2 &= (ds/uc)^{1/2}, \\ b_1/b_2 &= s/c. \end{aligned} \tag{12}$$

The quantity β given by (9) is treated as a free parameter since it involves the unknown quantities a_3' and b_3'. We diagonalize $\mathcal{M}$ of (10) and find the ν_e-ν_μ mixing angle θ to be given by

$$\tan 2\theta = -2(d/s)^{1/2}\{6(dc/us)^{1/2} + [(c/s) - \beta]/(a - \beta)\}^{-1}, \tag{13}$$

where

$$a = (uc/ds)^{1/2}. \tag{14}$$

The two neutrino mass eigenvalues m_1 and m_2 obtained by diagonalizing Eq. (10) are related to this mixing angle by

$$\Delta m^2 \equiv m_2^2 - m_1^2 = (m_1 + m_2)^2 \cos 2\theta, \tag{15}$$

which, of course, shows $\Delta m^2 \to 0$ for maximal mixing ($\theta = \pi/4$).

We now choose standard values for the quark mass ratios, namely,

$$(d/s)^{1/2} \simeq \theta_c = 0.22,$$
$$c/s \cong 10. \qquad (16)$$
$$d/u = 1.5.$$

Maximal mixing, i.e., $\theta = \pi/4$, occurs for $\beta \sim 2.89$. For mixing between $0.1 < \sin^2 2\theta < 1$ (probably the interesting range for observable oscillations) β lies within 1% of this value. (Of course, there always remains the residual $1/3\ \theta_c$ coming from the charged lepton sector.) This demonstrates how accurately the parameters of the Majorana sector are to be tuned if we are to achieve observable mixing.

We now consider the case of three generations, still with two Higgs. The obvious generalizations,[10] which follow from a discrete symmetry, are

$$M_1^U = \begin{pmatrix} 0 & \mathbf{10} & 0 \\ \mathbf{10} & \mathbf{126} & 0 \\ 0 & 0 & \mathbf{10} \end{pmatrix} = \begin{pmatrix} 0 & (uc)^{1/2} & 0 \\ (uc)^{1/2} & c & 0 \\ 0 & 0 & t \end{pmatrix}, \qquad (17)$$

$$M_2^U = \begin{pmatrix} 0 & \mathbf{10} & 0 \\ \mathbf{10} & 0 & \mathbf{126} \\ 0 & \mathbf{126} & \mathbf{10} \end{pmatrix} = \begin{pmatrix} 0 & (uc)^{1/2} & 0 \\ (uc)^{1/2} & 0 & (ct)^{1/2} \\ 0 & (ct)^{1/2} & t \end{pmatrix}, \qquad (18)$$

with similar forms for M^D in each case. The first case obviously requires tuning as above for large mixing. We find that (18) leads to very small ν_e-ν_μ mixing even if we allow the possibility of tuning the parameters. Unfortunately both of these forms imply the experimentally unacceptable relation

$$t = (uc/ds)^{1/2}\, b, \tag{19}$$

if we limit ourselves to only two Higgs in generating the above mass matrices. We have examined various alternatives and are lead to the following conjecture: there is no SO(10) model with three generations and two Higgs scalars (for generating fermion masses), derivable from discrete symmetries, which implies correct fermion mass ratios. Nevertheless, we regard the above examples as sufficient to make our essential point: Observable ν_e-ν_μ mixing can only be achieved in SO(10) with an unreasonable adjustment of parameters.

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REFERENCES

1. H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93**, 193 (1975); H. Georgi, **Particles and Fields**, 1974 (APS/DPF Williamsburg) ed. C. E. Carlson (AIP, New York, 1975).
2. M. Gell-Mann, P. Ramond, and R. Slansky, **Supergravity**, eds. P. van Nieuwenhuizen and D. Z. Freedman (North-Holland Publishing Company, 1979), p. 315; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number of the Universe, edited by O. Sawada and A. Sugamoto, KEK, 1979 (unpublished).
3. K. Milton and K. Tanaka, Phys. Rev. **D23**, 2087 (1981); S. Hama, K. Milton, S. Nandi, and K. Tanaka, Phys. Lett. **97B**, 221 (1980).
4. T. Goldman and G. J. Stephenson, Phys. Rev. **D24**, 236 (1981); T. Yanagida and M. Yoshimura, Phys. Lett. **97B**, 99 (1980); K. Kanaya, Prog. Theor. Phys. **64**, 2278 (1980).
5. F. Reines, H. W. Sobel, and E. Pasierb, Phys. Rev. Lett. **45**, 1307 (1980); V. A. Lyubimov, E. G. Novikov, V. Z. Nozik, E. F. Tretyakov, V. S. Kosik, Phys. Lett. **94B**, 268 (1980); J. N. Bahcall **et al.**, Phys. Rev. Lett. **45**, 945 (1980); D. Silverman and A. Soni, Phys. Rev. Lett. **46**, 467 (1981).
6. S. L. Glashow, Harvard preprint HUTP-80/A089.
7. K. Milton and K. Tanaka, Ohio State University preprint DOE/ER/0/545-309.
8. E. Witten, Phys. Lett. **91B**, 81 (1980).

9. We only regard mass relations which are independent of p, that is ratios involving only quark masses or only lepton masses, as valid.

10. H. Fritzsch, Phys. Lett. **73B**, 317 (1978).